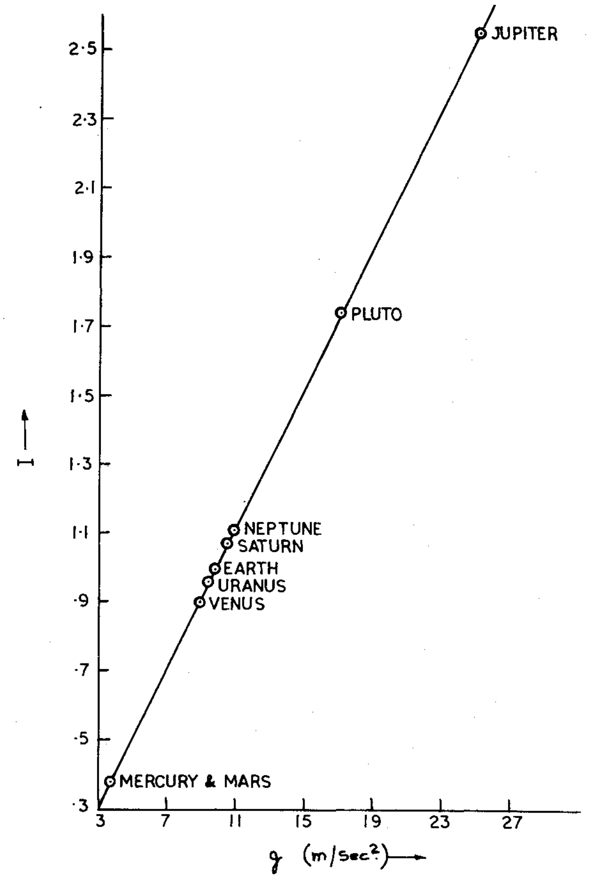

Fig. 1 Dependence of sedimentation current on gravity for different plantes. (ordinate in units of I for Earth).

Sedimentation Potential and Rate in Gravitational Fields

The irreversible thermodynamics of sedimentation phenomena have been discussed by de Grott,[4] and Rastogi and Misra.[5] When insoluble solid particles are allowed to fall under gravity, potential energy is converted into electrical energy and a sedimentation current flows. Moreover, in such a system, when an electric field is applied the particles move, giving rise to electrophoresis. The phenomenological relations can be written as

$$I = L_{11}\, E + L_{12} g \tag{26}$$

$$J = L_{21} E + L_{22} g \tag{27}$$

where E is the electric field and g is the gravitational field per unit mass. I is the electric current density and J is the material flux related to V_e, the electrophoretic velocity, in the following manner:

$$J = m(1 - \rho_2/\rho_1) \cdot V_e \tag{28}$$

where m is the mass of particles suspended between the two electrodes. ρ_1 and ρ_2 are the density of solid and liquid, respectively. When $E = 0$

$$I = L_{12} g \qquad J = L_{22} g \tag{29}$$

The value of I and J on the moon would be roughly one sixth of that on earth. Furthermore, under conditions of zero gravity

$$(I/E)_{g=0} = L_{11}\,(J/E)_{g=0} L_{21} \tag{30}$$

In Fig. 1, the sedimentation current for a typical system is plotted against g for various planets in units of sedimentation current for earth.

References

[1] Carruthers, J.R., and Grasso, M., "Studies of Floating Liquid Zones in Simulated Zero Gravity," *Journal of Applied Physics,* Vol. 43, Feb. 1972, p. 436.

[2] Guggenheim, E.A., *Thermodynamics,* North-Holland Pub. Co., Amsterdam, 1957.

[3] Buckingham, A.S., *The Laws and Application of Thermodynamics,* Macmillan, New York, 1964.

[4] deGroof, S.R. and Mazur, P., *Non-equilibrium Thermodynamics,* North-Holland Pub. Co., Amsterdam, 1962, p. 160.

[5] Rastogi, R.P., and Misra, B.M., "Cross-Phenomenological Coefficients, Part-7. Electrophoresis and Sedimentation Potential," *Transactions, Faraday Soc.,* Vol. 63, 1967, pp. 584.

Quasi-Steady Gas Phase Assumption for a Burning Droplet

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Nomenclature

C $= C = c_F h^0_F + c_P h^0_P$
c_F $= w_F / w_O N_O$
c_P $= w_P N_P / w_O N_O$
C_p = total heat capacity at constant pressure, cal/g° K
D = diffusivity, cm^2/sec
h^0_i = specific formation enthalpy for species i, cal/g
k_f = the Arrhenius term, cm^3/mole sec
N = stoichiometric number of moles
p = pressure, atm
P = nondimensional pressure
r = radial coordinate, cm
R = radius of the droplet, cm
$\tilde{R}$ = gas constant, cal/g° K
t = time, sec
T = temperature, ° K
u = velocity, cm/sec
w_i = molecular weight of species i, g/mole
y = nondimensional coordinate
Y_i = mass fraction for species i
z = nondimensional time
β = characteristic frequency, 1/sec
γ = the isentropic gas coefficient
θ = nondimensional temperature
λ = coefficient of heat conductivity, cal/cm° Ksec
ρ = density, g/cm^3
τ = characteristic time, sec

Subscripts

chem = chemical
d = droplet
F = fuel
g = gas
O = O_2
P = products
p = pressure
ref = reference
0 = initial conditions

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NATURAL phenomena are all, strictly speaking, transient. However, a given process may behave on the average in a quasi-steady manner even though momentarily it is transient. Whenever this is so, it is advisable to treat the process as quasi-steady, thereby introducing a mathematical simplification. Because of this simplification, many authors have made the quasi-steady assumption for problems in different fields.

A large number of results obtained in the field of droplet combustion are based upon the assumption that the gas field behaves in a quasi-steady manner. However, this assumption is introduced usually without adequate justification. Therefore, it is felt that the discussion of the possibility of realistically making a quasi-steady assumption for the gas phase deserves particular attention. The method presented in this work can be followed by anyone interested in checking whether the quasi-steady assumption is valid for a particular problem. The numerical evaluations are performed here for a typical average behavior at low pressures, but the interested person should re-evaluate the numbers for his particular problem. The assumptions relevant to the present work can all be found in the available literature.[1]

There are two categories of conditions under which the quasi-steady assumption made for the gas phase holds:

a) The gas phase relaxes much faster than the liquid phase. Then the uncoupling between the liquid phase, which is considered unsteady, and the gas phase, which we would like to consider as quasi-steady, is realistic. The characteristic time for the liquid phase is

$$\tau_d = \frac{D_d}{(-dR/dt)^2_{ref}}$$

Similarly,

$$\tau_g = \frac{D_g}{(u_g - dR/dt)^2_{ref}}$$

By using the quasi-steady continuity equation at the droplet surface, we obtain

$$\tau_g/\tau_d = (D_g/D_d)\,(\rho_g/\rho_d)^2$$

Assume $\rho_d/\rho_g \sim 1000$, $D_d = 1.5$ mm^2/sec, and $D_g = 15$ mm^2/sec at 298° K and 1 atm.[2] Then $\tau_g = 10^{-5}\tau_d$, which proves that the gas phase and the liquid phase can indeed be uncoupled.

b) The gas phase is quasi-steady by itself because the characteristic time for a change of the dependent variables is much larger than the characteristic times for diffusion, convective, and chemical processes.

Consider the case of a droplet burning under an imposed pressure $p = p(0)e^{-\beta t}$. The unsteady energy equation for the gas phase can be written in a nondimensional form as follows

$$\frac{\partial \theta_g}{\partial z} + \left(u_g - \beta\frac{dR}{dz}\right)\frac{1}{R\beta}\frac{\partial \theta_g}{\partial y} - \frac{\lambda_g}{\rho_g C_{pg}}\frac{1}{\beta R^2}\frac{1}{y^2}\frac{\partial}{\partial y}\left(y^2\frac{\partial \theta}{\partial y}\right) - \frac{p(0)}{T(0)\rho_g C_{pg}}\frac{dP}{dz} - \frac{k_f}{w_F}\frac{C}{\beta C_{pg}T(0)}\frac{p(0)}{\tilde{R}(0)T(0)}\frac{\tilde{R}(0)}{\tilde{R}}\frac{P}{\theta}Y_F Y_O = 0 \tag{1}$$

where $z = \beta t$, $\theta = T/T(0)$, $y = r/R$, and $P = p/p(0)$. We made the assumptions that λ_g is a constant and that the chemical processes in the gas phase are represented by a one-step reaction which is of second order. The possibility of having a regressing droplet surface was also considered in Eq. (1).

Now, in order for the quasi-steady assumption to be valid, the terms containing time derivatives must be negligible with respect to the other terms of Eq. (1). In particular, there will be two categories of conditions, one for eliminating $\partial\theta_g/\partial z$, the other for neglecting dP/dz. If all these conditions are satisfied, the quasi-steady assumption will be valid for the gas phase.

1) Conditions for eliminating $\partial\theta_g/\partial z$

a)
$$\left(u_g - \beta\frac{dR}{dz}\right)\frac{1}{R\beta} >> 1 \tag{2}$$

If one neglects dR/dz with respect to R (which is a realistic approximation), Eq. (2) can be written $\beta << u_g/R$. An estimate of u_g/R is easy to make from the quasi-steady continuity equation. Assuming that $\rho_d/\rho_g \sim 1000$, Eq. (2) can be written as a condition upon the characteristic times:

$$\tau_p >> \tau_d/334 \tag{3}$$

The pressure dependence of the previous relation is expressed by

$$\tau_p >> \tau_{d_p}/334 \tag{4}$$

where p is expressed in atmospheres.

b)
$$(\lambda_g/\rho_g C_{pg})(1/\beta R^2) >> 1 \tag{5}$$

If one assumes $Le = 1$, this condition becomes $\beta << D_g/R^2$. From the quasi-steady droplet analysis,[3] one can evaluate

$$\frac{D_g}{R(t=0)^2} = \frac{1}{2\tau_d \ln\{1+B\}}$$

A good numerical estimate for $\ln\{1+B\}$ at a pressure of 1 atm is 1/6. Therefore, the condition expressed by Eq. (5) is translated into a condition upon characteristic times:

$$\tau_p >> \tau_d/3 \tag{6}$$

The dependence upon pressure is given by

$$\tau_p >> \tau_d p/3 \tag{7}$$

where p is expressed in atmospheres.

c)
$$\frac{k_f}{W_F}\frac{C}{\beta C_{pg}T(0)}\frac{p(0)}{\tilde{R}(0)T(0)}\frac{\tilde{R}(0)}{\tilde{R}}\frac{P}{\theta} >> 1 \tag{8}$$

We know that $P \sim 1$ and, if we assume $\tilde{R}(0)/\tilde{R} \sim 1$ and $\theta \sim 10$, then Eq. (8) can be expressed in terms of characteristic times:

$$\tau_p >> 10\tau_{\text{chem}} \tag{9}$$

The pressure dependence is obtained from

$$\tau_p >> 10\tau_{\text{chem}_{p=1\text{atm}}}\frac{1}{p} \tag{10}$$

where p is given in atmospheres.

2) Conditions for eliminating dP/dz:

a)
$$\frac{p(0)}{T(0)\rho_g C_{pg}} << \left(u_g - \beta\frac{dR}{dz}\right)\frac{1}{R\beta} \tag{11}$$

Now

$$\frac{P(0)}{T(0)\rho_g C_{pg}} = \frac{p(0)}{T(0)}\frac{\tilde{R}T(0)}{p(0)}\frac{\theta_g}{P}\frac{1}{C_{pg}} = \frac{\tilde{R}}{C_{pg}}\frac{\theta_g}{P} = \frac{\gamma-1}{\gamma}\frac{\theta_g}{P}$$

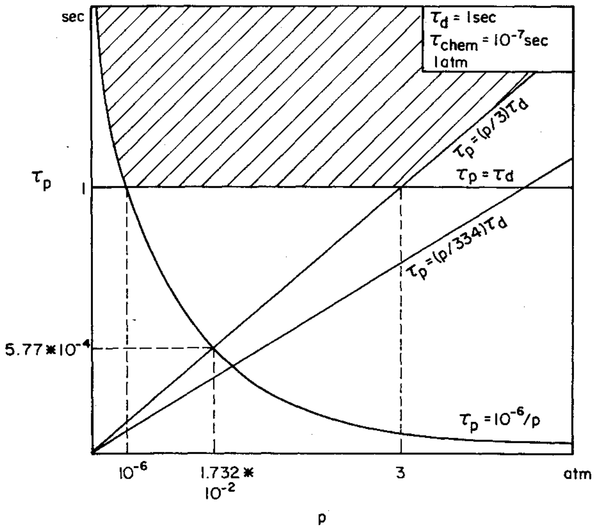

Fig. 1 Sketch representing the domain of validity for the quasi-steady gas field assumption.

assuming that the relationship $\tilde{R}/C_{pg}=(\gamma-1)/\gamma$ holds. Therefore,

$$\frac{p(0)}{T(0)\rho_g C_{pg}} \sim \frac{0.4}{1.4}\frac{10}{1} \sim 3 \tag{12}$$

Then Eq. (11) yields

$$\tau_p >> \tau_d/112 \tag{13}$$

There is no pressure dependence in this relationship.

$$b) \quad \frac{p(0)}{T(0)\rho_g C_{pg}} << \frac{\lambda_g}{\rho_g C_{pg}} \frac{1}{\beta R^2} \tag{14}$$

By use of Eqs. (12) and (6) one finds

$$\tau_p >> \tau_d \tag{15}$$

and no pressure dependence.

$$c) \quad \frac{p(0)}{T(0)\rho_g C_{pg}} << \frac{k_f}{w_F} \frac{C}{\beta C_{pg} T(0)} \frac{p(0)}{\tilde{R}(0)T(0)} \frac{P}{\theta} \frac{\tilde{R}(0)}{R} \tag{16}$$

Combining Eqs. (12) and (9) yields

$$\tau_p >> 30\tau_{\text{chem}} \tag{17}$$

without pressure dependence.

If Eqs. (4), (6), (9), (13), (15), and (17) are satisfied by the nonsteady problem, then the nonsteady situation can be treated as quasi-steady.

Figure 1 shows how one can find a region of quasi-steady behavior in the (p, τ_p) plane for a given droplet of a fixed size. The figure is based upon the previous six inequalities, and the numerical evaluations were obtained for

$$\tau_{\text{chem}_{/\text{atm}}} = 10^{-7}\text{sec and } \tau_d = 1 \text{ sec}$$

corresponding to droplets of 1-mm diam.[3] The domain in which the quasi-steady assumption is acceptable is covered with oblique lines in Fig. 1. This region is quite large and is certainly in the range of interest for the low pressure range in Diesel engines. For the purpose of applying this theory to combustion in engines or rockets, τ_d should be taken much smaller because in these cases the diameter of the droplets is at least one order of magnitude smaller than 1 mm.[1] As the value of τ_d decreases, the domain of validity for the quasi-steady assumption becomes larger.

References

[1] Williams, F. A., *Combustion Theory*, Addison-Wesley, Reading, Mass., 1965.

[2] American Petroleum Institute, *Petroleum Refining*, Division of Refining, New York, N. Y., 1966.

[3] Hall, A. R. and Diederichsen, J., "An Experimental Study of the Burning of Single Drops of Fuel in Air at Pressures Up to Twenty Atmospheres," *4th Symposium International on Combustion*, Williams and Wilkins Co., Baltimore, Md., 1953, pp. 837-346.

Flutter of Thermally Buckled Finite Element Panels

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Introduction

THE versatile finite element method has recently been extended to the panel flutter problem with noteworthy success. Olson[1,2] first developed the concept of aerodynamic matrix and formulated several plate finite elements for the flutter analysis of rectangular and delta wings. Similar work was conducted by Kariappa[3,4] et al. In those developments, the linearized supersonic flow theory approximated by the first-order frequency of oscillation was used. Yang[5] later used a numerical integration technique to incorporate in the finite element formulation an exact linearized flow theory that includes the higher-order frequencies.

In this study, the finite element formulation given in Ref. 1 is extended to treat the flutter problem of a semi-infinite panel which is buckled into large deflections due to aerodynamic heating. The postbuckling behavior of the panel is predicted by a piecewise linear incremental procedure[6] combined with coordinate transformation at every step. The results obtained from illustrative examples are compared with alternative solutions[7-9] with reasonable agreement.

Formulation

The semi-infinite panel is defined by the following parameters: chord length a; thickness h; mass density m (mass per unit area); bending rigidity $D=Eh^3/12(1-\nu^2)$; nondimensional deflection $W=w/a$; and thermal expansion coefficient α. One surface of the panel is exposed to a supersonic flow with velocity U, Mach number M, and mass density ρ (mass per unit volume). The flow thus has a pressure $q=1/2$ ρU^2.

The panel is supported at both edges with the trailing edge free to roll in the plane of the panel. The trailing edge is also restrained elastically in the plane direction of the panel with elastic constant k (force per unit spanwise length). The panel is assumed to be subjected to uniform aerodynamic heating with a temperature rise of ΔT degrees. The heating produces inplane compressive stress N_x (force per unit spanwise length) and causes buckling when $N_x=\pi^2 D/a^2$. The rise in tem-

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